hence the right inequality holds.
Using the Cauchy-Schwarz inequality, we obtain

$$\begin{aligned}\left(\sum_{k=1}^{n} x_k\right)^2 &= \left(\sum_{k=1 \text{ cyclic}}^{n} 1\cdot\frac{x_k+x_{k+1}}{2}\right)^2\\ &\leq (1^2+1^2+\cdots+1^2)\left(\sum_{k=1 \text{ cyclic}}^{n}\frac{(x_k+x_{k+1})^2}{4}\right)\\ &= n\left(\sum_{k=1 \text{ cyclic}}^{n}\frac{(x_k+x_{k+1})^2}{4}\right).\end{aligned}$$

Now, the inequality $\frac{(x_k+x_{k+1})^2}{4} \leq \frac{x_k^2+x_kx_{k+1}+x_{k+1}^2}{3}$ holds as being equivalent to $(x_k - x_{k+1})^2 \geq 0$. Thus,

$$\left(\sum_{k=1}^{n} x_k\right)^2 \leq n\left(\sum_{k=1 \text{ cyclic}}^{n}\frac{x_k^2+x_kx_{k+1}+x_{k+1}^2}{3}\right)$$

and this gives the left inequality at once.

Also solved by the proposer.

58. Corrected. *Proposed by Arkady Alt, San Jose, California, USA.* Let P be arbitrary interior point in a triangle ABC and r be inradius. Prove that

$$\frac{a^2}{d_a(P)}+\frac{b^2}{d_b(P)}+\frac{c^2}{d_c(P)} \geq 36r$$

if $d_a(P)$, $d_b(P)$ and $d_c(P)$ are the distances from the point P to the sides BC, CA and AB respectively.

Solution 1 by Omran Kouba, Higher Institute for Applied Sciences and Technology, Damascus, Syria. (The original statement has r^2 instead of r, and it is clearly not correct, because it is not homogeneous.)
Let $s=(a+b+c)/2$. For $q \geq 1$ we have

$$\frac{2s}{3}=\frac{a+b+c}{3} \leq \left(\frac{a^{\frac{q+1}{2}}+b^{\frac{q+1}{2}}+c^{\frac{q+1}{2}}}{3}\right)^{\frac{2}{q+1}}.$$

Equivalently

$$3^{1-q}2^{q+1}s^{q+1} \leq \left(a^{\frac{q+1}{2}}+b^{\frac{q+1}{2}}+c^{\frac{q+1}{2}}\right)^2.$$

Now, let F_a, F_b, F_c and F represent the areas of the triangles PBC, PCA, PAB and ABC respectively. Clearly, we have $2F_a = ad_a(P)$, $2F_b = bd_b(P)$ and $2F_c = cd_c(P)$. Consequently, using Cauchy-Schwarz inequality, we have

$$\begin{aligned}3^{1-q}2^{q+1}s^{q+1} &\leq \left(a^{\frac{q+1}{2}}+b^{\frac{q+1}{2}}+c^{\frac{q+1}{2}}\right)^2\\ &\leq \left(\frac{a^{q+1}}{2F_a}+\frac{b^{q+1}}{2F_b}+\frac{c^{q+1}}{2F_c}\right)(2(F_a+F_b+F_c))\\ &\leq 2F\cdot\left(\frac{a^q}{d_a(P)}+\frac{b^q}{d_b(P)}+\frac{c^q}{d_c(P)}\right)\end{aligned}$$

Thus, since $F = sr$ we obtain

$$3^{1-q}2^q s^q \leq r \cdot \left(\frac{a^q}{d_a(P)} + \frac{b^q}{d_b(P)} + \frac{c^q}{d_c(P)} \right)$$

Finally, using the well-known inequality: $s \geq 3\sqrt{3}r$ we obtain

$$3^{1+q/2}2^q r^{q-1} \leq \frac{a^q}{d_a(P)} + \frac{b^q}{d_b(P)} + \frac{c^q}{d_c(P)}.$$

For $q = 1$ we get

$$6\sqrt{3} \leq \frac{a}{d_a(P)} + \frac{b}{d_b(P)} + \frac{c}{d_c(P)}$$

and for $q = 2$ we get

$$36r \leq \frac{a^2}{d_a(P)} + \frac{b^2}{d_b(P)} + \frac{c^2}{d_c(P)}.$$

which is the announced inequality.

59. *Proposed by Marcel Chiriţă, Bucharest, Romania.* Solve in real numbers the system

$$\left.\begin{array}{r} 2^x + 2^y = 12 \\ 3^x + 4^z = 11 \\ 3^y - 4^z = 25 \end{array}\right\}.$$

Solution 1 by Omran Kouba, Higher Institute for Applied Sciences and Technology, Damascus, Syria. Let $\alpha = \ln 2 / \ln 3 \in (0, 1)$ so that the second equation is equivalent to $2^x = (11 - 4^z)^\alpha$ and the third one is equivalent to $2^y = (25 + 4^z)^\alpha$ the first equation is then equivalent to $(11 - 4^z)^\alpha + (25 + 4^z)^\alpha = 12$. Now, consider $f(t) = (11 - t)^\alpha + (25 + t)^\alpha$ for $t \in [0, 11)$. Clearly

$$f'(t) = \alpha \left((25 + t)^{\alpha-1} - (11 - t)^{\alpha-1} \right) < 0$$

because $\alpha - 1 < 0$ and $11 - t \leq 11 < 25 + t$ for $t \in [0, 11)$. It follows that f is strictly decreasing on $[0, 11)$, and since $f(2) = 9^\alpha + 27^\alpha = 2^2 + 2^3 = 12$ we conclude that 2 is the only solution of the equation $f(t) = 12$ that belongs to $[0, 11)$. It follows that the equation $f(4^z) = 12$ has $z = 1/2$ as unique solution. But then $2^x = (11 - 4^z)^\alpha = 2^2$ and $2^y = (25 + 4^z)^\alpha = 2^3$. Therefore, the proposed system has a unique real solution which is $(x, y, z) = (2, 3, \frac{1}{2})$.

Solution 2 by Adnan Ali (student), Mumbai, India.
Eliminating 4^z, we have the system

$$\begin{array}{c} 2^x + 2^y = 12 \\ 3^x + 3^y = 36 \end{array}$$

We prove that the only solutions for the above system are $\{x, y\} = \{2, 3\}$. The proof is based on the following claim: Fix positive constants a, b and $k > 1$, then the equation in t:

$$t^k + (a - t)^k = b,\ 0 \leq t \leq a$$

may have at most two solutions. Indeed, we let $f(t) = t^k + (a - t)^k - b$ and observe that since $k - 1 > 0$, the derivative $f'(t) = k(t^{k-1} - (a - t)^{k-1})$ is negative for $t < \frac{a}{2}$, vanishes at $t = \frac{a}{2}$, and is positive for $t > \frac{a}{2}$. Hence $f(t)$ is strictly decreasing from 0 to $\frac{a}{2}$ and strictly increasing from $\frac{a}{2}$ to a, and our claim follows.
Now we let $t = 2^x$, $r = 2^y$ and $k = \frac{\ln 3}{\ln 2} > 1$ so that the system is now